OCTOGON Mathematical Magazine
Vol. 33. No. 2B. October, 2025.

OCTOGON MATHEMATICAL MAGAZINE
Vol. 33, No.2, October 2025, pp 1275-1293
Print: ISSN 1222-5657, Online: ISSN 2248-1893
https://web.uni-miskolc.hu/∼matsefi/Octogon/

Vibration of centricexcentric trilobic evolutive supermathematical systems

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Dedicated to Themistocles M. Rassias on the occasion of his 75th Birthday

1 INTRODUCTION: CENTRICEXCENTRIC TRILOBES

We purported in the paper " ABOUT VIBRATIONS WITH SUPERMATHEMATICAL TWISTS" , in the motto, that the Vibrations are the "left bread"right of scientists: a vibration → an equation, another vibration → another equation, another equation → another vibration, and so on, and in this paper, we reaffirm our assertion (even if the first author does not consider himself a scientist!)

Consequently:

• The vibrations of the linear systems correspond to the circular centric functions (FCC) sin and cos. Well known!

• After the circular excentric supermathematical functions (FSM - CE) cex and sex, the vibrations of systems of excentric nonlinear elastic characteristics also appeared [2],[3], less known. Not by the authors' fault;

• After defining the quadrilobic excentric FSM (FSM - QE) coq and siq, the quadrilobic vibrations also appeared [4];

•After defining the trilobic excentric FSM (FSM - TE) cet and set, the trilobic mechanical vibrations also appeared.

[25]Received: 27.08.2025
2010 *Mathematics Subject Classification.* 13B02.
Key words and phrases. Supermathematics.

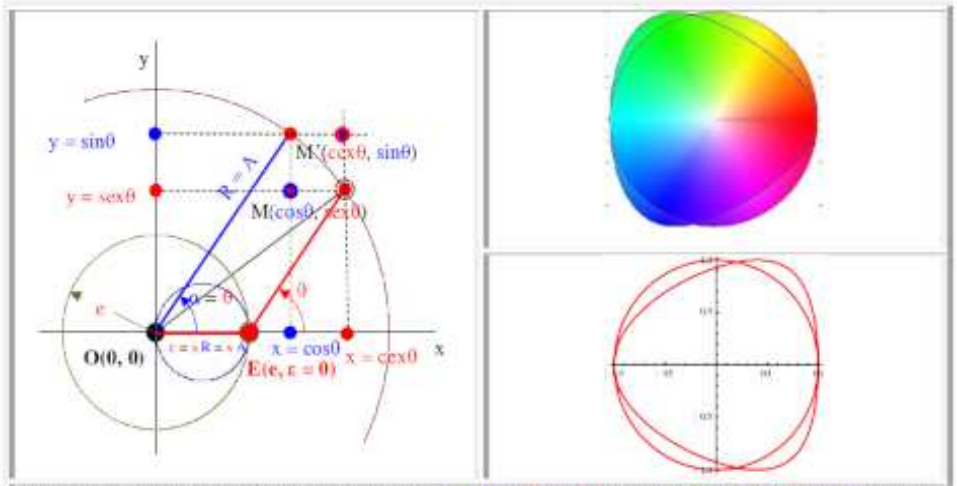


Fig. 1,a The diagram of of the vibration sizes of the **centricexcentric evolutive supermathematical** systems ◀ and the trajectory of the representative points M ▶▲ and M' ▶▼

It is normal, not knowing if it is, at the same time, good, as with the appearance of the evolutive supermathematical functions (FSM- EV) the corresponding vibrations that we call centricexcentric evolutive vibrations (CoE) also appear, because the characteristic point rotates on these closed curves, whose projections, on two mutually perpendicular axes, evolve on centricexcentrice closed curves, also called trilobes, since it has 3 lobes.

The explanatory diagram is presented in figure 1,a and their graphs in figure 1,c, and are expressed by equations:

$$\begin{cases} x = cex\theta = \cos\{\theta - \arcsin[s.\sin(\theta - \varepsilon)]\} = cex\theta \\ y = \sin\theta \end{cases},$$

$$CoE\ trilobes\ of\ cex \rightarrow (Fig.1, b\ ◀) \tag{1}$$

$$\begin{cases} x = \cos\theta \\ y = sex\theta = \sin\{\theta - \arcsin[s.\sin(\theta - \varepsilon)]\} = sex\theta \end{cases},$$

$$CoE\ trilobes\ of\ sex \rightarrow (Fig.1, b\ ▶) \tag{2}$$

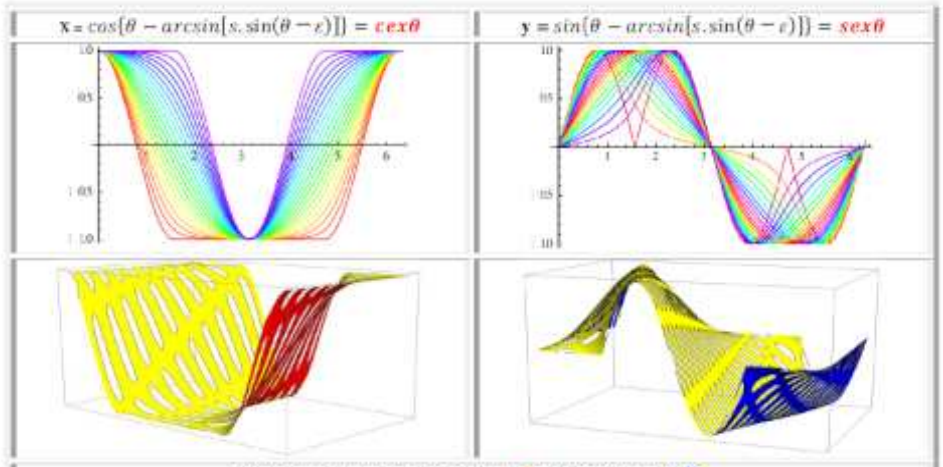
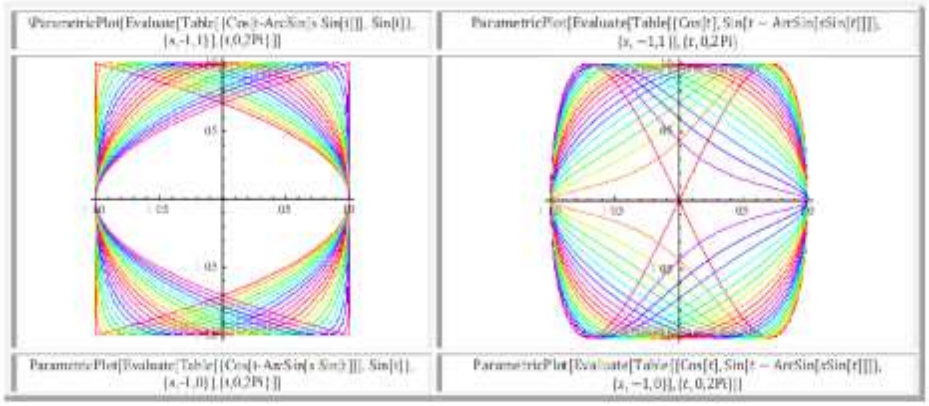
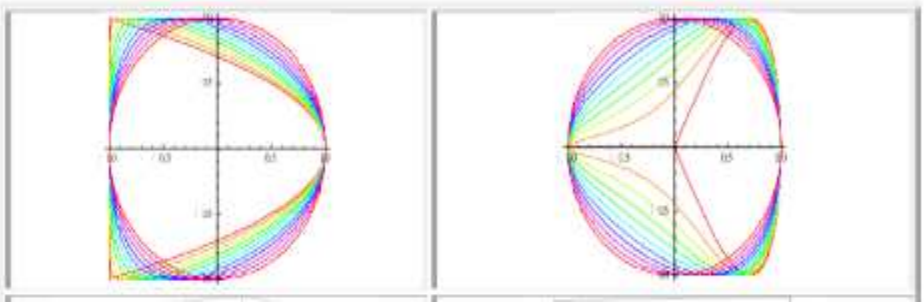


Fig. 1,b The graphs of **x** from (1) ◀ and of **y** from (2) ▶





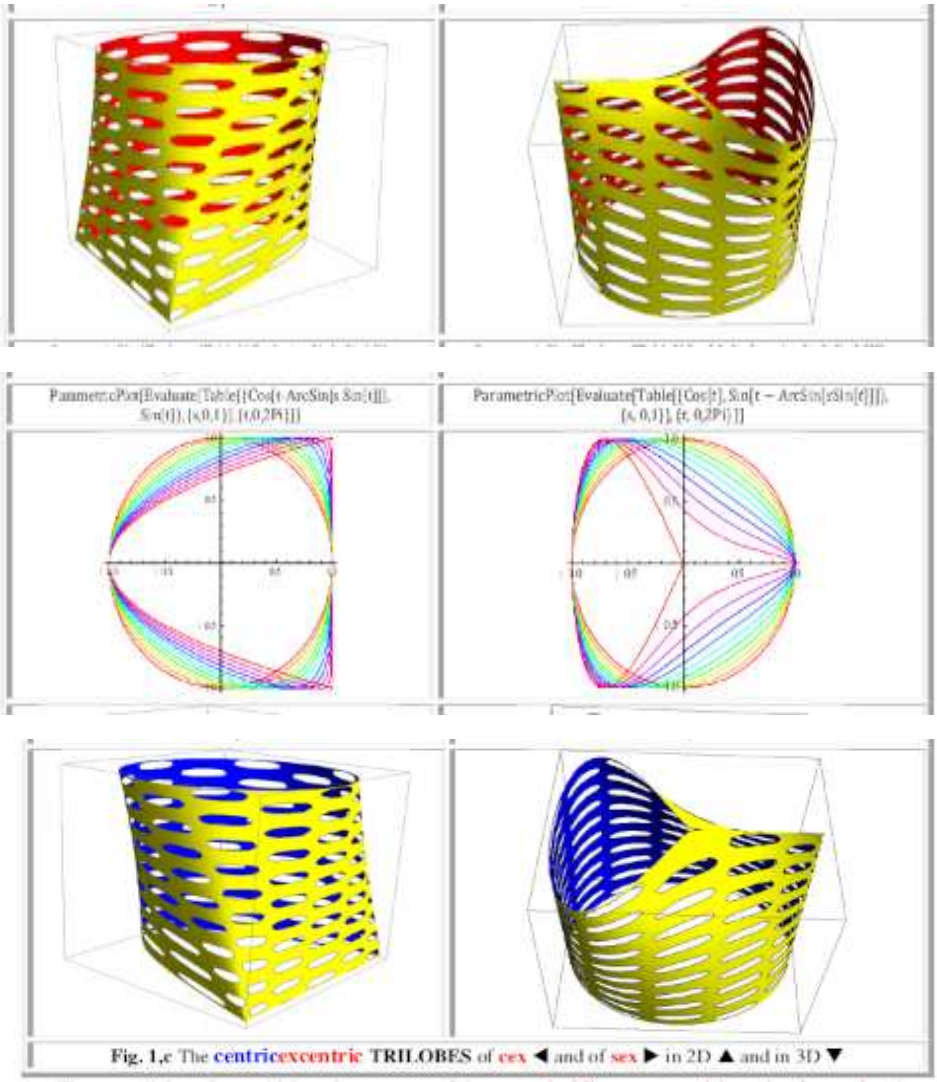


Fig. 1,e The centricexcentric **TRILOBES** of cex ◀ and of sex ▶ in 2D ▲ and in 3D ▼

The true trilobes, those which at the extremes of the numerical linear excentricity s, i.e. for s = 0 and s =+1 degenerate into a circle or into a triangle, respectively, in the present case a perfect rectangular triangle, and for s=- 1 degenerate in a "left triangle"right which we call SM excentric rectangular triangle (Fig. 1,d ◀ ▲) or imploded triangle, which has two obtuse angles and a third, right angle; all around the same point: the center of the circle O(0,0).

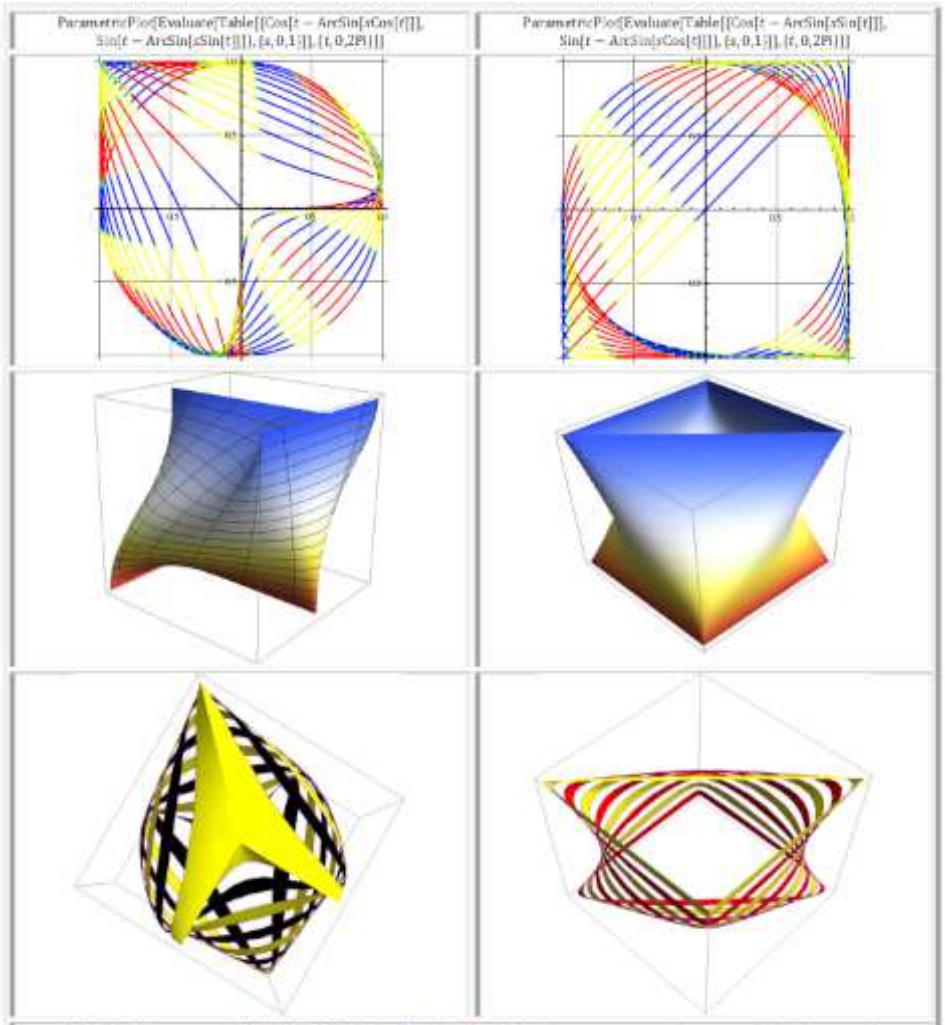


Fig. 1,d The **excentric TRILOBES** S ◀ and C ▶ of functions x = **cex** and y = **sex** simultaneously but of numerical linear excentricities **s** and/or different angular **ε**, $(\varepsilon_x = \frac{\pi}{2}, \varepsilon_y = 0)$ ◀ $(\varepsilon_x = 0, \varepsilon_y = \frac{\pi}{2})$ ▶

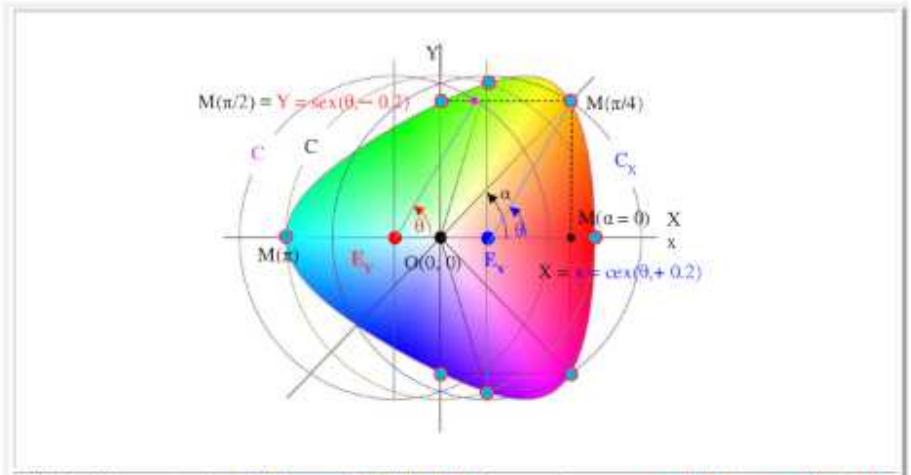
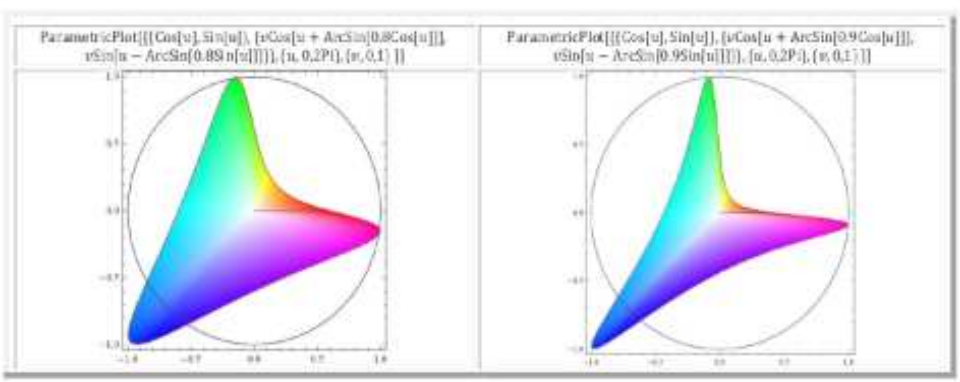


Fig. 1,e The construction of **SM excentric trilobe** (TE) of parametric eq. **x = cex(θ; 0.2); y = sex(θ, − 0.2)**

The triangle is defined as a three-dimensional geometric figure. Therefore, the graphs presented in figure 1,f may be called triangles, even if, for $s < 1$, they have curvilinear sides. Two of the sides, L_1 and L_2, are equals between them (so it can also be called isosceles triangle) and equals with the radius of the circle, in this case of the unit circle R = 1. The third side is equal to the half-diagonal of the square of side 2, i.e. $L_3 = \sqrt{2}$ can be oriented to any of the corners of the square, as can be seen in figure 1,f.

If the sum of the inner angles of a triangle is $\pi = 180°$ and of the external angles is 5π, then of SM excentric triangles is $0°$ inside the triangles, 2π in its center and in the origin of the coordinate axis system, and 6π in the vertices of SM excentric triangle.



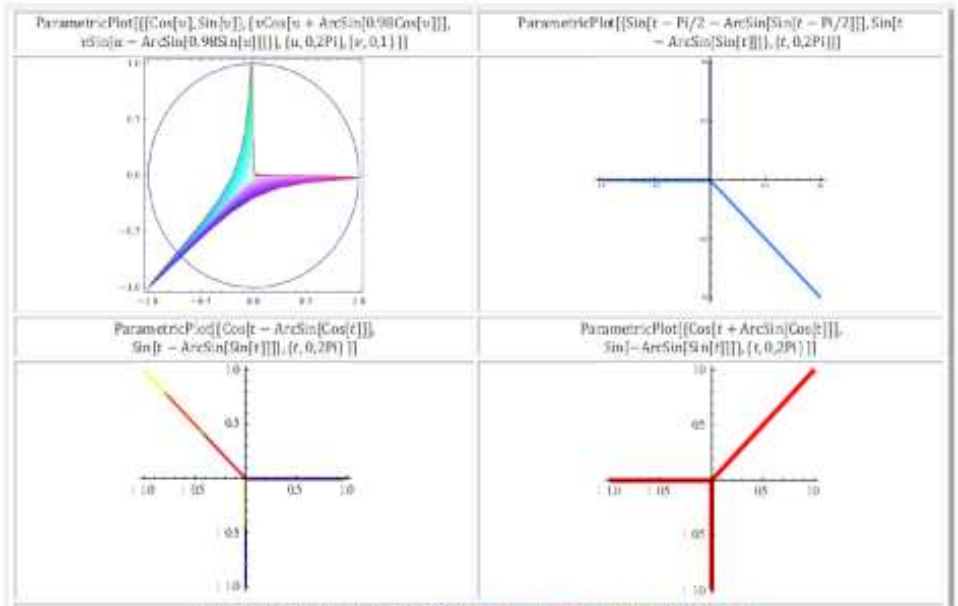


Fig. 1,f The construction of **SM** (**TE**) **excentric trilobe S** of **s = 1**, respectively, of **excentric triangles** or of **imploded trilobes**

A clearer construction of an excentric trilobe has the explanatory scheme from figure 1,e and is of parametric equations

$$M \begin{cases} x = cex\,(\theta, s_x = +0,2, \varepsilon = 0) = cex\,(\theta, s_x = +0,2, \varepsilon = 0) \\ y = sex\,(\theta, s_y = -0,2, \varepsilon = 0) = sex\,(\theta, s_y = -0,2, \varepsilon = \pi) \end{cases} \tag{3}$$

where the two excenters are $S_x\,(s_x = +0.2, \varepsilon_x = 0)$ and $S_y\,(s_y = -0.2, \varepsilon_y = 0) \equiv S_x\,(s_y = +0.2, \varepsilon_y = \pi)$.

2 THE DIFFERENTIAL EQUATION OF THE VIBRANT TRILOBIC CENTRICEXCENTRIC SYSTEMS

By rotating the excentric half-line d^+ around the excenters E_x and E_y or S_x and S_y with an angular constant speed Ω (Fig. 1,a), the representative point M will rotate on the trilobes with a variable linear speed v and a variable angular speed ω, expressed by FSM- CE excentric derivative of θ, dexθ, of equation:

$$dex\theta = \frac{d\alpha}{d\theta} = \frac{d\alpha}{dt} \cdot \frac{dt}{d\theta} = \frac{\omega}{\Omega} = 1 - \frac{s.\cos\,(\theta - \varepsilon)}{\sqrt{1 - s^2 \sin^2\,(\theta - \varepsilon)}} \tag{4}$$

such that the variable angular speed ω, of the representative point M on the excentric trilobe, will be:

$$\omega = \Omega.dex\theta \tag{5}$$

and its speed, for a circle of radius R on which the excentric trilobes are defined, will be

$$v = R.\Omega.dex\theta = R.\Omega\left[1 - \frac{s.\cos(\theta - \varepsilon)}{\sqrt{1 - s^2\sin^2(\theta - \varepsilon)}}\right] \tag{6}$$

For simplicity, from hereon we take into consideration R = 1, for excentric trilobes on unit / trigonometric circles CU(O, R=1) and an angular constant speed $\Omega = 1$ around the excenters Sx and Sy. If the shift in the x and y directions is given by the functions (1) or (2), then the speeds of the projections in these two directions with graphics from figure 2,a are:

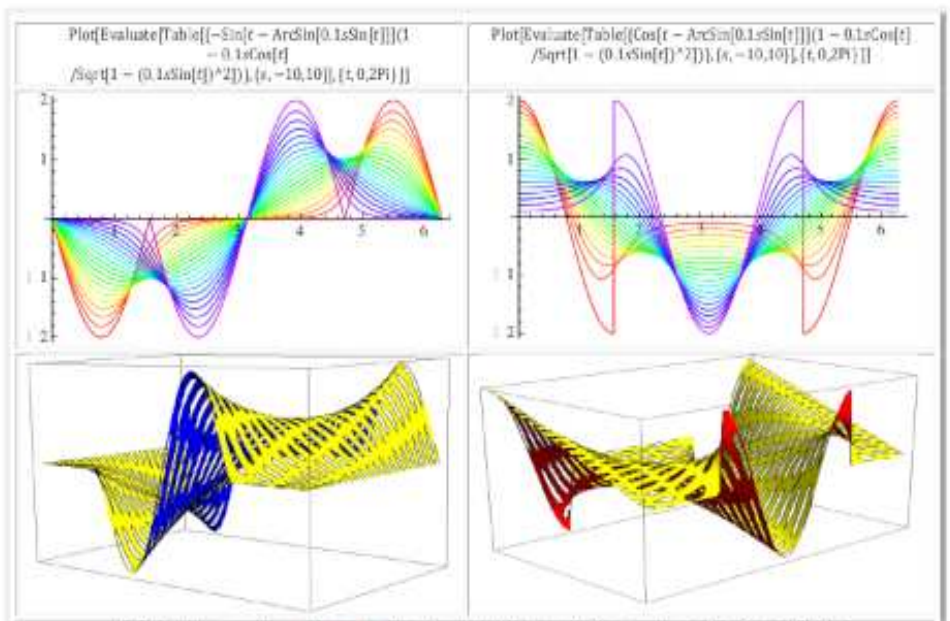
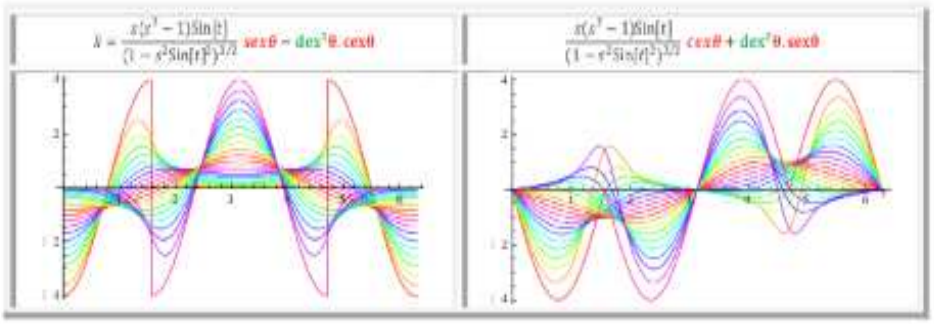


Fig. 2,a The graphics of speeds of vibrant **trilobic evolutive** systems **(1)** ◀ **and (2)** ▶



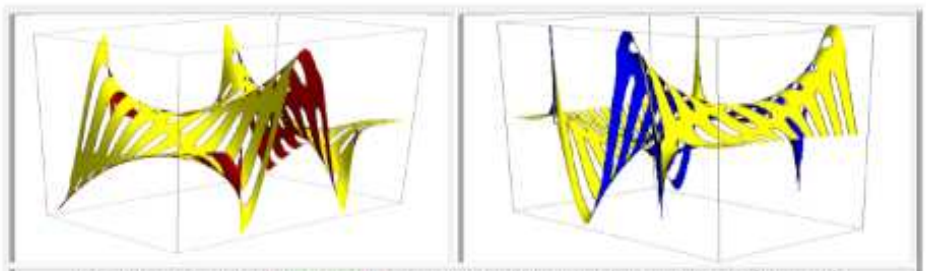


Fig. 2,b The graphics of accelerations of vibrant **trilobic evolutive** systems **(1) ◀ and (2) ▶**

$$\begin{cases} \dot{x} = \dot{cex}\theta = d\frac{\cos(\theta - \arcsin[s.\sin(\theta-\varepsilon)])}{dt} = -dex\theta.sex\theta \\ \dot{y} = \dot{\sin}\,\theta = \cos\theta \end{cases},$$

$$CoE\ trilobes\ of\ cex \rightarrow (Fig.2, a\ ◀) \tag{7}$$

$$\begin{cases} \dot{x} = \dot{\cos}\,\theta = -\sin\theta \\ \dot{y} = \dot{sex}\theta = d\frac{\sin(\theta - \arcsin[s.\sin(\theta-\varepsilon)])}{dt} = -dex\theta.cex\theta \end{cases},$$

$$CoE\ trilobes\ of\ sex \rightarrow (Fig.2, a\ ▶) \tag{8}$$

and, therefore, the accelerations will be:

$$\begin{cases} \ddot{x} = \dot{cex}\theta = d\frac{2\cos\{\theta - \arcsin[s.\sin(\theta-\varepsilon)]\}}{dt^2} = -\frac{d}{dt}\left[dex\theta.sex\theta\right] = \\ = -\left[dex\theta.sex\theta + dex\theta.sex\theta\right] \\ \ddot{x} = \frac{s\left(s^2-1\right)\sin[t]}{\left(1-s^2\sin[t]^2\right)^{3/2}}sex\theta - dex^2\theta.cex\theta \\ \ddot{y} = \ddot{\sin}\,\theta = \dot{\cos}\,\theta = -\sin\theta \end{cases} \tag{9}$$

$$\begin{cases} \ddot{x} = \ddot{\cos}\,\theta = -\cos\theta \\ \ddot{y} = sex\theta = d\frac{\sin\{\theta - \arcsin[s.\sin(\theta-\varepsilon)]\}}{dt} = dex\theta.cex\theta = \\ = \frac{s\left(s^2-1\right)Sin[t]}{\left(1-s^2Sin[t]^2\right)^{3/2}}cex\theta + dex^2\theta.sex\theta \end{cases} \tag{10}$$

with the graphics from figure 2,b.
The necessary and sufficient condition for the equations (1) and (2) to be solutions of the vibrant trilobic system, therefore to form a fundamental system of solutions, is that the Wronski's determinant, or the corresponding wronskian, not to be identical null on the interval $[0, 2\pi]$.
The Wronski's determinant is:
For the first case:

$$(1) : \begin{cases} x = cex\theta \\ y = \sin\theta \end{cases} \rightarrow (7) \begin{cases} x' = -dex\theta sex\theta \\ y' = \cos\theta \end{cases}$$

such that the wronskian will be

$$W(x) = \begin{vmatrix} x & y \\ \dot{x} & \dot{y} \end{vmatrix} = \begin{vmatrix} cex\theta & \sin\theta \\ -dex\theta sex\theta & \cos\theta \end{vmatrix} =$$

$$= \begin{vmatrix} \cos[\theta - \arcsin[s.\sin\theta]] & \sin\theta \\ \left(1 - sCos[t]/Sqrt\left[1-(sSin[t])^2\right]\right)Cos[t - ArcSin[sSin[t]]] & \cos\theta \end{vmatrix} =$$

$$= Det\begin{bmatrix} Cos[t - ArcSin[sSin[t]]] & Sin[t] \\ -\left(1 - sCos[t]/Sqrt\left[1-(sSin[t])^2\right]\right)Sin[t - ArcSin[sSin[t]]] & Cos[t] \end{bmatrix} =$$

$$= Cos[t]\,Cos[t - ArcSin[sSin[t]]] + Sin[t]\,Sin[t - ArcSin[sSin[t]]] -$$

$$-\frac{sCos[t]\,Sin[t]\,Sin[t - ArcSin[sSin[t]]]}{\sqrt{1 - s^2Sin[t]^2}} = Sin[t]\,Sin[t - ArcSin[sSin[t]]] +$$

$$+Cos[t]\left(Cos[t - ArcSin[sSin[t]]] - \frac{sSin[t]\,Sin[t - ArcSin[sSin[t]]]}{\sqrt{1 - s^2Sin[t]^2}}\right) \tag{11}$$

with the graphics from figure 3.

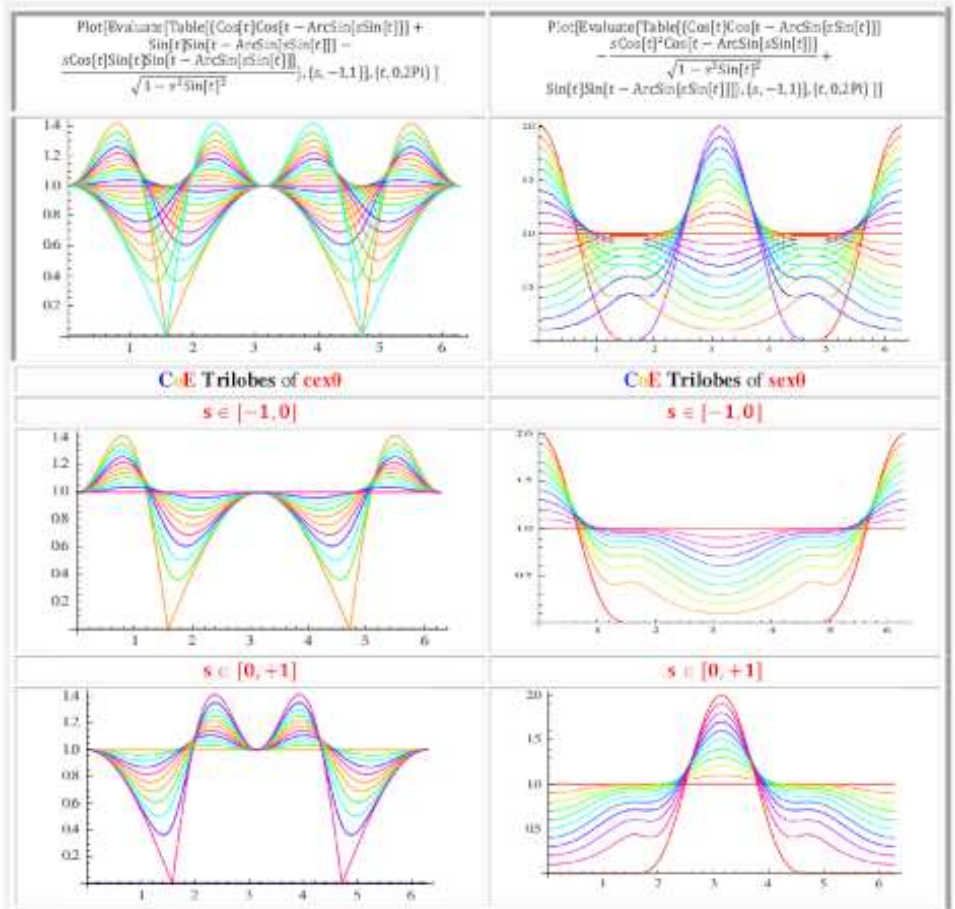


Fig. 3 The graphics of Wronski Matrices

As can be seen from the graphics from figure 3, the Wronski matrices are positive and greater than zero ($W \geq 0$), in both cases, such that both equations (1) and (2) form a fundamental system of solutions of vibrations of trilobic evolutive systems.

Therefore, there is a linear differential equation, of a dynamic technical system, of nonlinear elastic characteristic, admitting these functions as a fundamental system of solutions.

These fundamental solutions are:

$$\begin{cases} Z_1 = C_1 cex\Omega t + C_2 \sin \Omega t \\ Z_2 = C_1 \cos \Omega t + C_2 sex\Omega t \end{cases} \rightarrow Fig.4 \tag{12}$$

where $C_1, C_2 \in R$ are constant and (13) is the general solution of the following differential equations:

The equations are:

$$\begin{vmatrix} z & x & y \\ \dot{z} & \dot{x} & \dot{y} \\ \ddot{z} & \ddot{x} & \ddot{y} \end{vmatrix} = 0 \tag{13}$$

$$\ddot{z} \begin{vmatrix} x & y \\ \dot{x} & \dot{y} \end{vmatrix} - \dot{z} \begin{vmatrix} x & y \\ \ddot{x} & \ddot{y} \end{vmatrix} + z \begin{vmatrix} \dot{x} & \dot{y} \\ \ddot{x} & \ddot{y} \end{vmatrix} = 0 \tag{14}$$

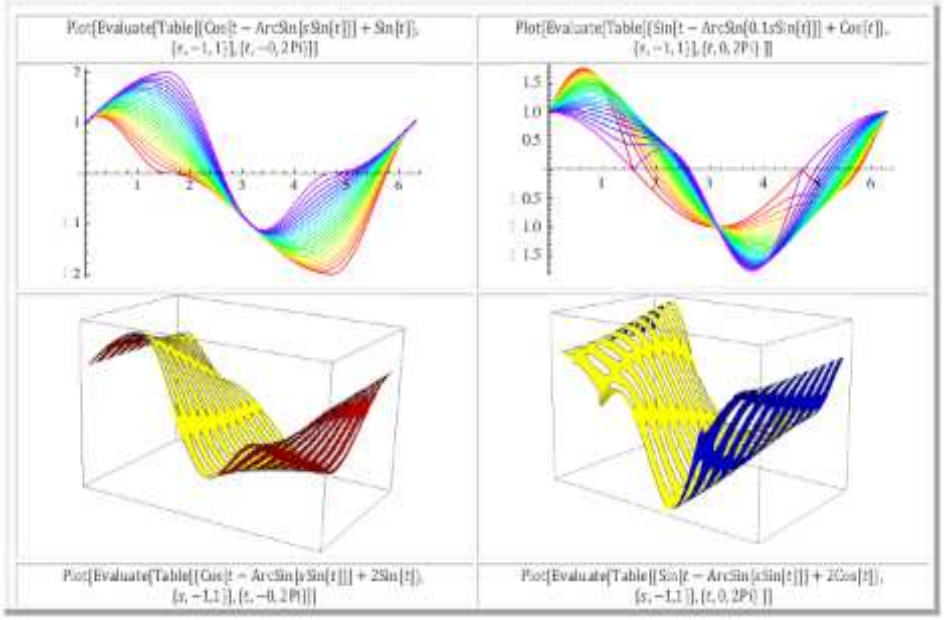
$$\begin{cases} \begin{vmatrix} z & cex\theta & \sin\theta \\ \dot{z} & -\omega.sex\theta & \Omega.\cos\theta \\ \ddot{z} & -3.sex\theta - \omega^2.cex\theta & -\Omega^2.\sin\theta \end{vmatrix} = 0 \\ \begin{vmatrix} z & \cos\theta & sex\theta \\ \dot{z} & -\Omega.\sin\theta & \omega.cex\theta \\ \ddot{z} & -\Omega^2.\cos\theta & -3.cex\theta - \omega^2.sex\theta \end{vmatrix} = 0 \end{cases} \tag{15}$$

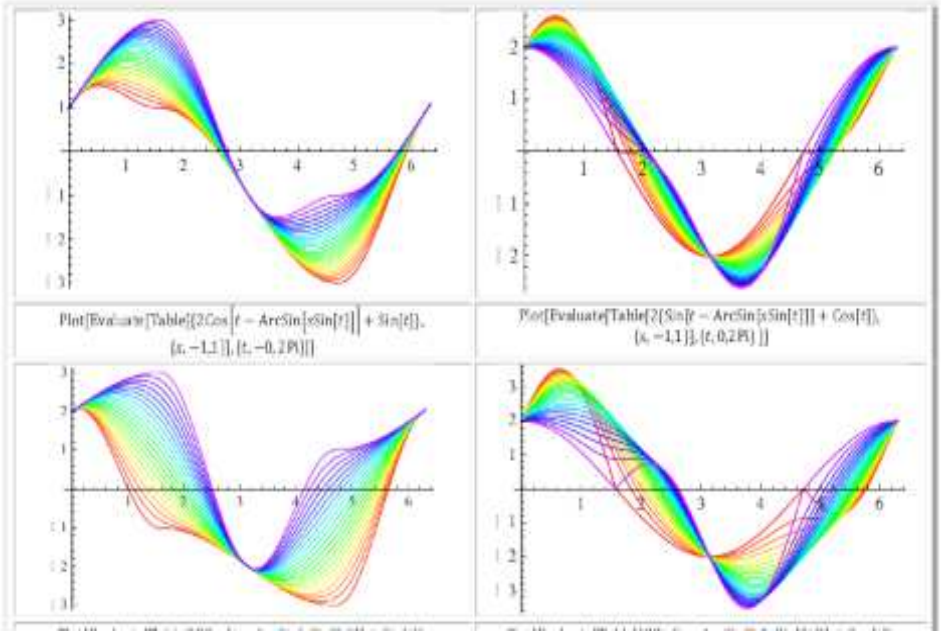
where

$$\begin{cases} \omega = \Omega.dex\theta = \Omega.dex\Omega.t, \ angular \quad speed \\ 3 = \dot{\omega} = -\dot{\omega}^2 \frac{s\left(1-s^2\right)\sin(\theta-\varepsilon)}{\left(1-s^2\sin^2(\theta-\varepsilon)\right)^{\frac{3}{2}}}, \ angular \quad acceleration \end{cases} \tag{16}$$

$$\begin{cases} \ddot{z} \begin{vmatrix} cex\theta & \sin\theta \\ -\omega.sex\theta & \Omega.\cos\theta \end{vmatrix} - \dot{z} \begin{vmatrix} cex\theta & \sin\theta \\ -3.sex\theta - \omega^2.cex\theta & -\Omega^2.\sin\theta \end{vmatrix} + \\ +z \begin{vmatrix} -\omega.sex\theta & \Omega.\cos\theta \\ -3.sex\theta - \omega^2.cex\theta & -\Omega^2.\sin\theta \end{vmatrix} \\ \ddot{z}\left(\Omega.\cos\theta.cex\theta + \omega.\sin\theta\right) - \dot{z}\left[-\Omega^2.\sin\theta.cex\theta + \sin\theta\left(3.sex\theta - \omega^2.cex\theta\right)\right] + \\ +z\left[\omega.\Omega^2.sex\theta.\sin\theta + \Omega.\cos\theta\left(3.sex\theta + \omega^2.cex\theta\right)\right] = 0 \end{cases} \tag{15'}$$

From (12) it is clear that the solutions are evolutive supermathematical centricexcentric functions [24] with graphics from figure 4.


Plot[Evaluate[Table[{Cos[t − ArcSin[sSin[t]]] + Sin[t]},
{s, −1, 1}], {t, −0, 2Pi}]]
Plot[Evaluate[Table[{Sin[t − ArcSin[0.1sSin[t]]] + Cos[t]},
{s, −1, 1}], {t, 0, 2Pi}]]
Plot[Evaluate[Table[{Cos[t − ArcSin[sSin[t]]] + 2Sin[t]},
{s, −1,1}], {t, −0, 2Pi}]]
Plot[Evaluate[Table[{Sin[t − ArcSin[sSin[t]]] + 2Cos[t]},
{s, −1,1}], {t, 0, 2Pi}]]


Plot[Evaluate[Table[{2Cos[t − ArcSin[sSin[t]]] + Sin[t]},
{s, −1,1}], {t, −0, 2Pi}]]
Plot[Evaluate[Table[{2{Sin[t − ArcSin[sSin[t]]] + Cos[t]},
{s, −1,1}], {t, 0,2Pi}]]

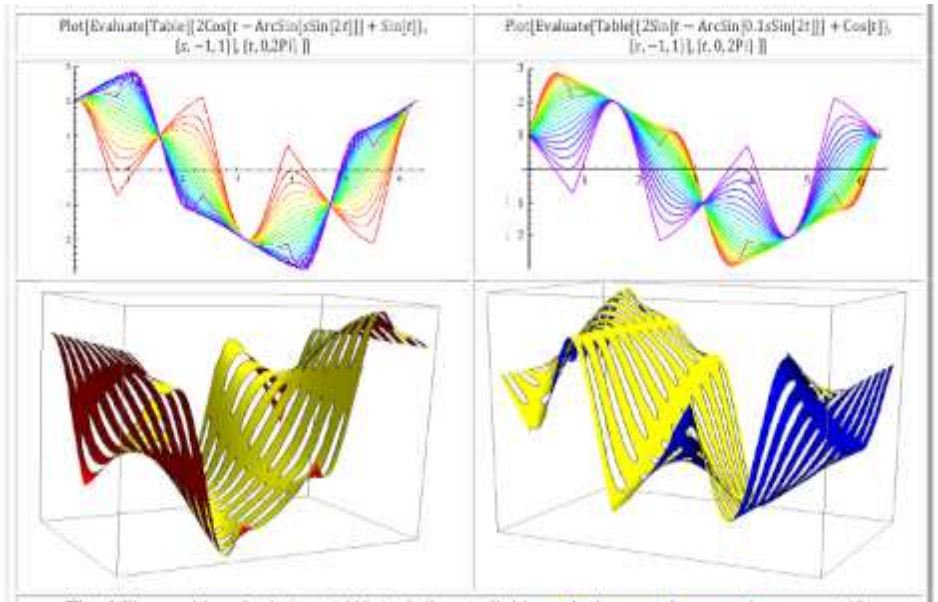
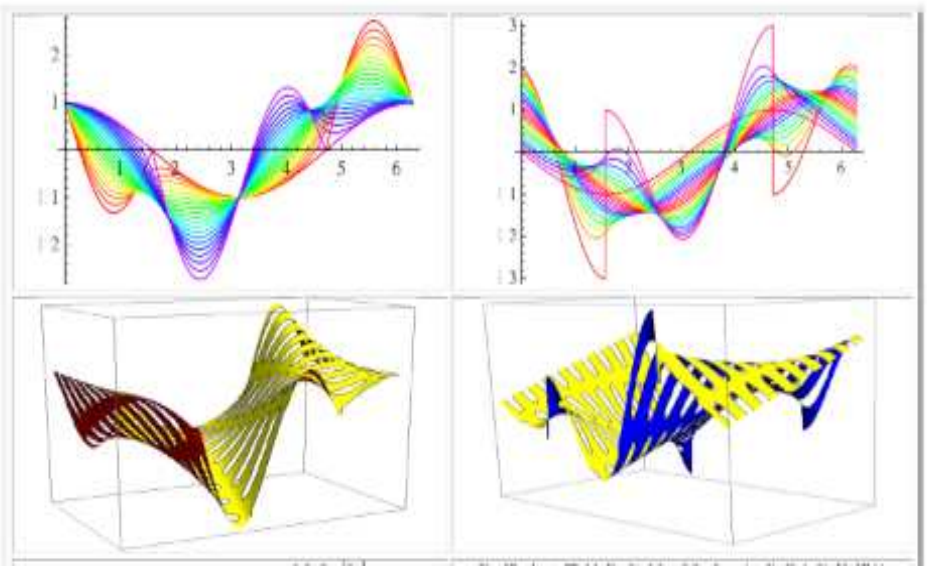


Fig. 4 The graphics of solutions (shifts) of vibrant trilobic **evolutive centric excentric** systems (12)



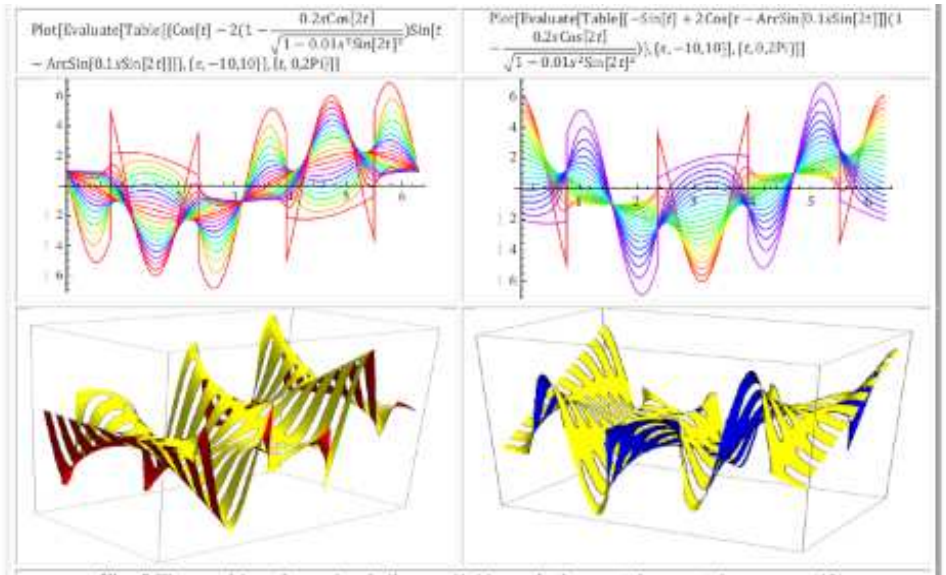


Fig. 5 The graphics of speeds of vibrant trilobic **evolutive centricexcentric** systems (12) of the first example (vibrations) from **figure 4**.

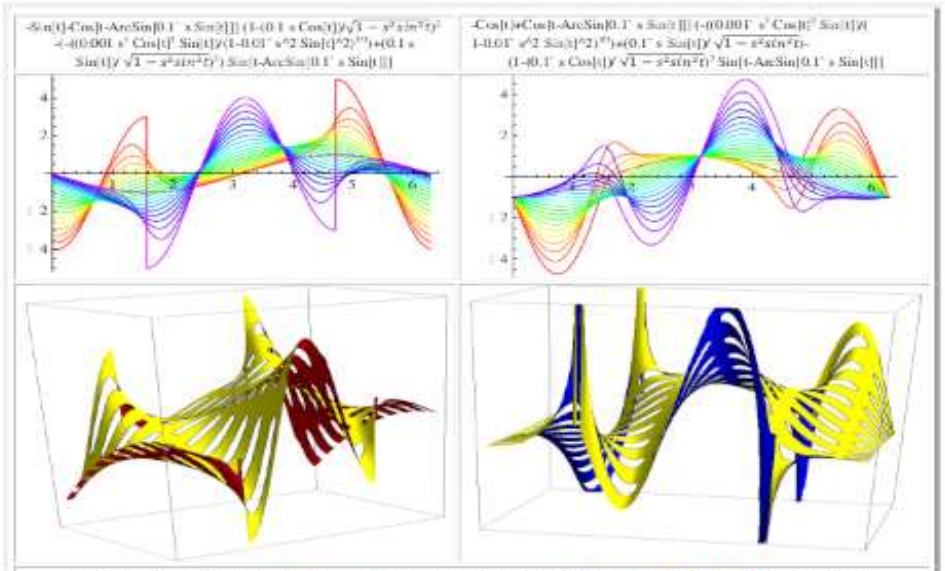
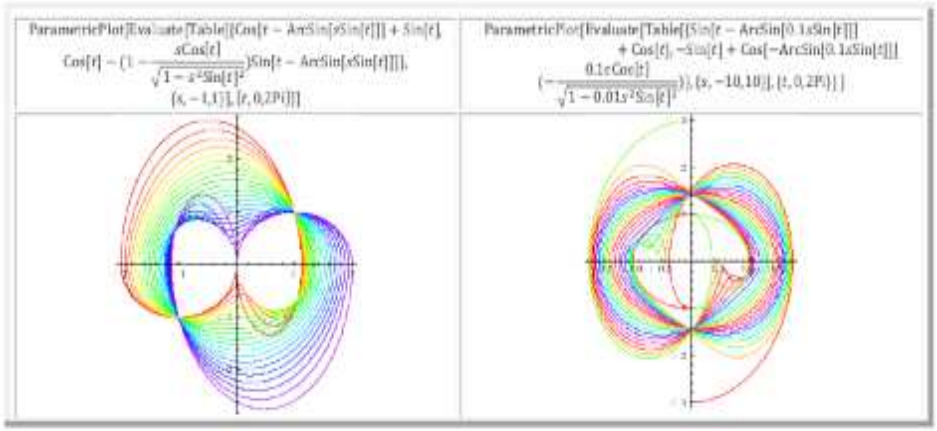


Fig. 6 The graphics of accelerations of vibrant trilobic **evolutive centricexcentric** systems (12) of the first example (vibrations) from **figure 4**.



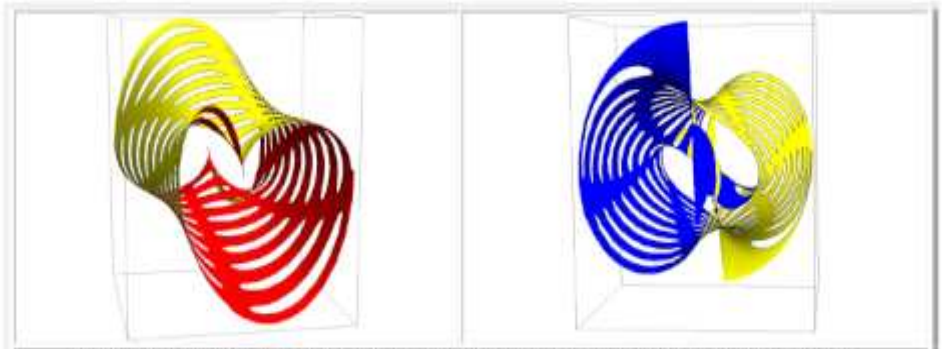
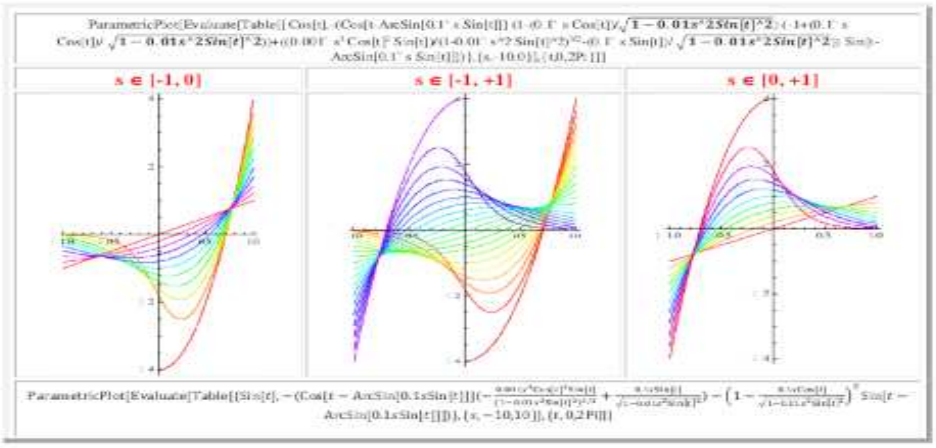


Fig. 7 The integral curves in the phase plane $\dot{x}(x)$ of the first example (vibrations) from **figure 4**.



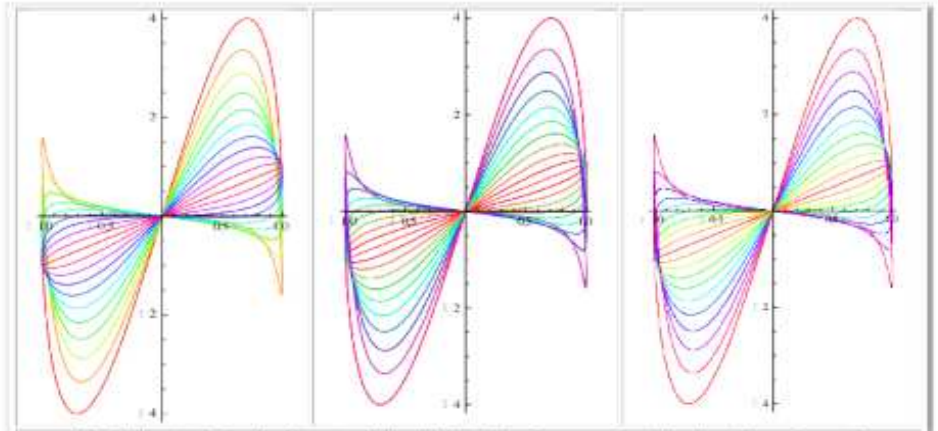


Fig. 6 The static elastic characteristics (SEC) of the first example (vibrations) from **figure 4**.

In figure 6 it can be noticed that for s = 0 all the nonlinear static elastic characteristics (SEC) become linear and pass through the origin.
Parametric equations of SEC are:

$$\begin{cases} x = \cos\theta \\ y = -\left(\frac{d^2}{d\theta^2} sex\theta\right) \end{cases} \tag{17}$$

$$\begin{cases} x = \sin\theta \\ y = -\left(\frac{d^2}{d\theta^2} cex\theta\right) \end{cases} \tag{18}$$

with graphics from figure 6.

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